

GSP-ExR: GSP Protocol with an Exclusive Right for Keyword Auctions

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ABSTRACT

We propose a keyword auction protocol called the Generalized Second Price with an Exclusive Right (GSP-ExR). In existing keyword auctions, the number of displayed advertisements is determined in advance. Thus, we consider adjusting the number of advertisements dynamically based on bids. In the GSP-ExR, the number of slots can be either 1 or K . When K slots are displayed, the protocol is identical to the GSP. If the value per click of the highest ranked bidder is large enough, then this bidder can exclusively display her advertisement by paying a premium. Thus, this pricing scheme is relatively simple and seller revenue is at least as good as the GSP. Also, in the GSP-ExR, the highest ranked bidder has no incentive to change the number of slots by over/under-bidding as long as she retains the top position.

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General Terms: Economics, Theory

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1. INTRODUCTION

Recently, billions of dollars are spent on keyword auctions run by search engines such as Google and Yahoo! [2, 4, 5]. In current keyword auctions, the number of displayed advertisements is determined in advance. In this paper, we consider adjusting the number of slots dynamically based on the bids. For example, if one bidder has a very high valuation per click compared to other bidders, this bidder is allowed to exclusively display her advertisement as long as she is willing to pay a premium. In this case, we can simultaneously obtain better social surplus as well as better revenue for the seller (search engine). When we apply the GSP protocol which is actually used as a keyword auction protocol, the highest ranked bidder can increase her utility by over-bidding to obtain the exclusive right, since she does not need to pay the premium. On the other hand, we can apply the well-known VCG protocol, which is guaranteed to satisfy strategy-proofness and Pareto efficiency. However, in

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our keyword auction setting, one serious limitation for using the VCG is that seller's revenue can be lower than the GSP, which would discourage search engines from introducing the VCG. Also, bidders have difficulty understanding this protocol, since determining the VCG payment is also quite complicated and not intuitive.

Thus, we develop a new auction protocol called the GSP with an Exclusive Right (GSP-ExR). The GSP-ExR can choose the optimal number of slots, either 1 or K . When K slots are displayed, the protocol is identical to the GSP. If the bid amount of the highest ranked bidder is large enough, then this bidder can exclusively display her advertisement by paying a premium. Therefore, this pricing scheme is relatively simple. Also, the amount of the premium is determined so that the highest ranked bidder has no incentive to change the number of slots by over/under-bidding. Our simulation results show that the GSP-ExR can improve both social surplus and seller's revenue.¹

2. MODEL

First, we illustrate a keyword auction model as follows. Assume a set of K slots on a specific keyword and a set of bidders $N = \{1, 2, \dots, n\}$ where $n \geq K$. In a keyword auction, each bidder submits a value per click on the advertisement for a keyword. We assume the value per click is independent of the rank of the slots. Formally, let v_i denote bidder i 's value per click, which is a private valuation for bidder i .

Search engine ranks the bidders in decreasing order based on the product of each bid and a click-through-rate (CTR). In advance the auctioneer determines each bidder's CTR, which is hidden from bidders. Many previously conducted studies on keyword auctions have assumed a separable CTR [1, 3, 5]. In this paper, we also use this assumption. Let $CTR_i(k, j)$ denote the probability that bidder i 's advertisement is clicked on the j -th highest slot among k slots. CTR is separable if we can represent it as follows: $CTR_i(k, j) = C_{k,j}q_i$. Here, q_i is a factor related to the quality of bidder i 's advertisement, estimated by the auctioneer. $C_{k,j}$ depends only on position (k, j) , while q_i is associated with bidder i . Naturally, we assume $C_{k,j}$ becomes larger when the position j is closer to the top: $\forall k, \forall j, C_{k,j} \geq C_{k,j+1}$. The number of slots can be either 1 or K in the GSP-ExR. We assume that the following conditions hold: $C_{1,1} > C_{K,1}$ and

¹Due to space limitation, we omit the simulation results in this paper.

$C_{1,1} < \sum_{j=1}^K C_{K,j}$. Most existing keyword auctions introduce reservation price r , i.e., a bidder's bid is considered only when $q_i b_i \geq r$ holds.

We assume that a bidder has a *quasi-linear utility*, which is calculated by the difference between bidder i 's true value per click and her payment. Furthermore, we define social surplus S_k when allocating k slots: $S_k = \sum_{j=1}^k C_{k,j} q_{(j)} b_{(j)}$. Here, we refer $q_{(j)} b_{(j)}$ to the j -th highest product. The optimal number of slots to maximize the obtained social surplus can be determined as follows: $k^* = \arg \max_{k \in \{1, K\}} S_k$.

3. GSP WITH AN EXCLUSIVE RIGHT

We illustrate our new keyword auction protocol called the GSP-ExR. The GSP-ExR is defined as follows:

- Each bidder submits a value per click of b_i .
- The auctioneer sorts $q_i b_i$ (that satisfies $q_i b_i \geq r$) in decreasing order. Then the auctioneer determines the number of displayed slots (either 1 or K) by comparing the obtained social surplus.
 - If $S_K \geq S_1$, the optimal number of slots k^* becomes K and K slots are allocated. To calculate the payments, we apply the GSP payment. In the GSP, bidder i , who is allocated the j -th highest slot, will pay a price per click of $p_i(K, j)$, which is defined as follows:

$$p_i(K, j) = \frac{q_{(j+1)} b_{(j+1)}}{q_i}. \quad (1)$$

- If $S_1 > S_K$, the optimal number of slots k^* becomes 1 and the highest ranked bidder has an exclusive right. Payment $p_{(1)}(1, 1)$ is calculated as follows:

$$p_{(1)}(1, 1) = \frac{1}{C_{1,1} q_{(1)}} (C_{K,1} q_{(2)} b_{(2)} + \sum_{j=2}^K C_{K,j} q_{(j)} b_{(j)}). \quad (2)$$

THEOREM 1. *In the GSP-ExR, the highest ranked bidder has no incentive to change the number of displayed advertisements from either 1 or K as long as she retains the top position.*

PROOF. For simplicity, assume that the quality of each bidder i 's advertisement is identical, i.e., $\forall i, q_i = 1$.

First, assume that when the highest ranked bidder declares a true valuation and the number of displayed slots is K , i.e., $S_K \geq S_1$. Then, we show that the highest bidder cannot increase her utility by over-bidding, i.e., her utility when she gets the exclusive right (denoted as $u_{(1)}(1, 1)$) is smaller than (or at most equal to) the utility when K slots are displayed (denoted as $u_{(1)}(K, 1)$).

$$\begin{aligned} u_{(1)}(1, 1) &= C_{1,1}(v_1 - \frac{1}{C_{1,1}}(C_{K,1}b_{(2)} + \sum_{j=2}^K C_{K,j}b_{(j)})) \\ &\leq C_{K,1}v_1 + \sum_{j=2}^K C_{K,j}b_{(j)} \\ &\quad - (C_{K,1}b_{(2)} + \sum_{j=2}^K C_{K,j}b_{(j)}) \quad (\because S_K \geq S_1) \\ &= C_{K,1}(v_1 - b_{(2)}) = u_{(1)}(K, 1). \end{aligned}$$

Similarly, we can show that when the highest ranked bidder declares a true valuation and the total number of displayed slots is 1, then the bidder cannot increase her utility even if she changes from 1 to K slots. As a result, the highest ranked bidder cannot improve her utility by changing the number of displayed slots. \square

In the GSP-ExR, we adjust the payment of the highest ranked bidder so that when $S_1 = S_K$, $u_{(1)}(1, 1) = u_{(1)}(K, 1)$ holds, i.e., she is indifferent whether she gets the exclusive right. On the other hand, in the GSP, the highest ranked bidder can increase her utility by over-bidding to obtain the exclusive right, since she does not need to pay the premium.

Note that when K slots are displayed, the highest ranked bidder might be able to increase her utility by moving down on the rank of displayed positions. Such a manipulation is possible in the original GSP and the GSP-ExR inherits this problem.

EXAMPLE 1. *Assume 2 slots and 2 bidders, 1 and 2. Suppose that each bidder has a value per click of $v_1 = 300$ and $v_2 = 200$. The advertisement quality's is identical, i.e., $q_1 = q_2 = 1$. A reservation price of r is set to 100. Suppose that the auctioneer determines $C_{k,j}$ as follows: $C_{1,1} = 0.5$ and $(C_{2,1}, C_{2,2}) = (0.4, 0.2)$. Assume that bidder i submits her true value v_i . We can calculate the social surplus of S_k as follows: $S_1 = 0.5 \times 300 = 150$ and $S_2 = 0.4 \times 300 + 0.2 \times 200 = 160$. As a result, the optimal number of slots is $k^* = 2$. The top position goes to bidder 1 and she pays 200 per click and gets utility $u_1(2, 1) = 0.4(300 - 200) = 40$.*

On the other hand, if bidder 1 submits a bid of 1,000, the total number of displayed slots becomes 1. Bidder 1 pays $(C_{2,1} + C_{2,2})b_{(2)}/C_{1,1} = (0.4 + 0.2)200/0.5 = 240$ and gets a utility of $0.5(300 - 240) = 30$, which is smaller than her original utility 40. Thus, bidder 1 cannot increase her utility by over-bidding to obtain the exclusive right.

4. CONCLUSIONS

We developed a new keyword auction protocol that dynamically adjust the number of slots, while in existing keyword auction protocols, the number of slots is determined in advance. Our newly developed GSP-ExR protocol, a modification of the GSP, simultaneously improves the obtained social surplus and the search engine revenue.

One limitation of the GSP-ExR is that the number of slots is limited to 1 or K . Our future works include developing strategy-proof keyword auction protocols with simple payment calculation methods, in which the auctioneer can select the number of slots more flexibly.

5. REFERENCES

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